Projectile Motion

Linear Motion

- A baseball is dropped from the top of a bridge 200m above the water.
- When does it hit the base? (water)
- What is its final velocity?

Given Variables

- A baseball is dropped from the top of a bridge 200m above the water.
- When does it hit the ground?
- What is its final velocity?

$Y_i = 200\ m$

$Y_f = 0\ m$

$a = -9.8\ m/s^2$

$v_i = 0\ m/s$

Solve for Time

$$y_f = y_i + v_y t + \tfrac{1}{2} a t^2$$

$$0 = 200 + 0 + \tfrac{1}{2}(-9.8)\, t^2$$

$$t = \pm 6.39s$$

Solve for Velocity

$$v_f^2 = v_i^2 + 2\,a\,d$$

$$v_f^2 = 0 + 2(-9.8)(-200)$$

$$v_f = \pm 62.61 \text{ m/s}$$

$\pm$ choose the one that makes sense

A little more complicated

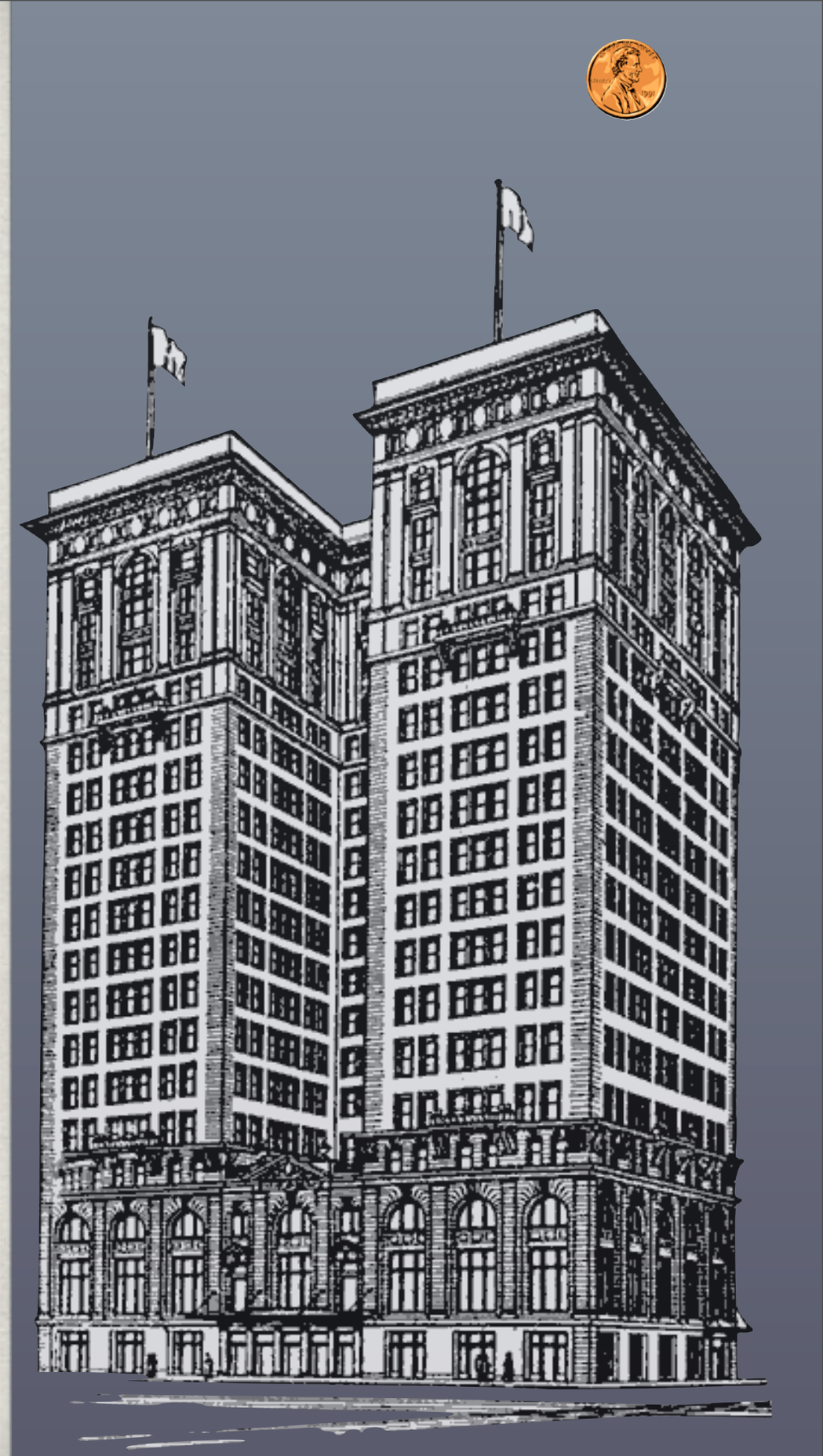
- A penny is thrown, straight up in the air, with an upward velocity of 15 m/s from the top of a 90 m building.
- How high does it go?
- When does it land?
- What is its final velocity?

Solve: Height at the Top

* $v_f^2 = v_i^2 + 2\,a\,d$
* $0 = 15^2 + 2(-9.8)d$
* $d = 11.5m$
* $Y_{max} = h = 101.5m$

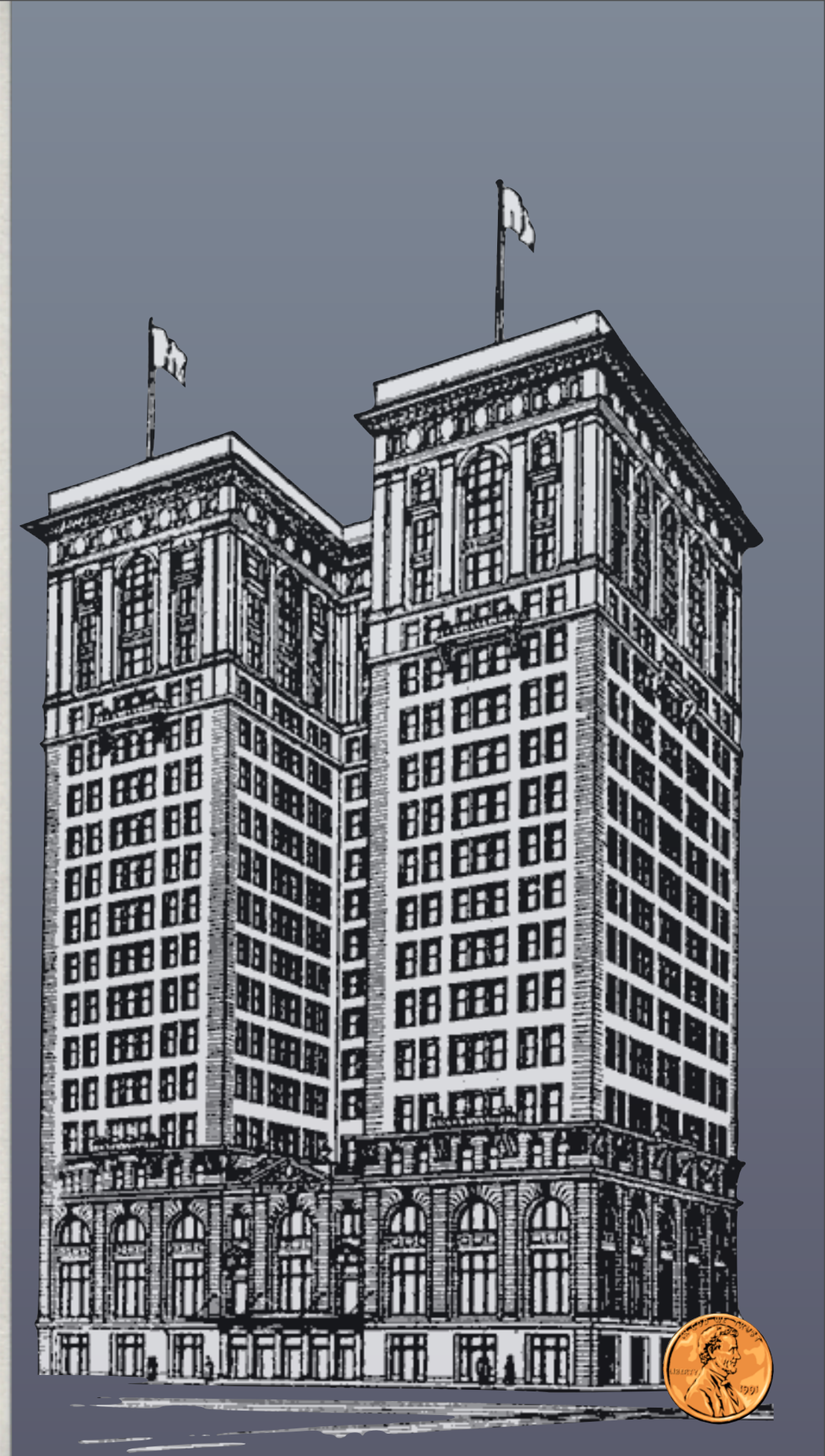
Solve:
Time at the Top

- $v_f = v_i + at$
- $0 = 15 + (-9.8)t$
- $t = 1.53\ s$



Solve:
Velocity at the Bottom

- $v_f^2 = v_i^2 + 2\ a\ d$
- $v_f^2 = 0 + 2(-9.8)(-101.5)$
- $v_f = \pm 43.91$ m/s
  - ± means you have to decide "up" or "down"



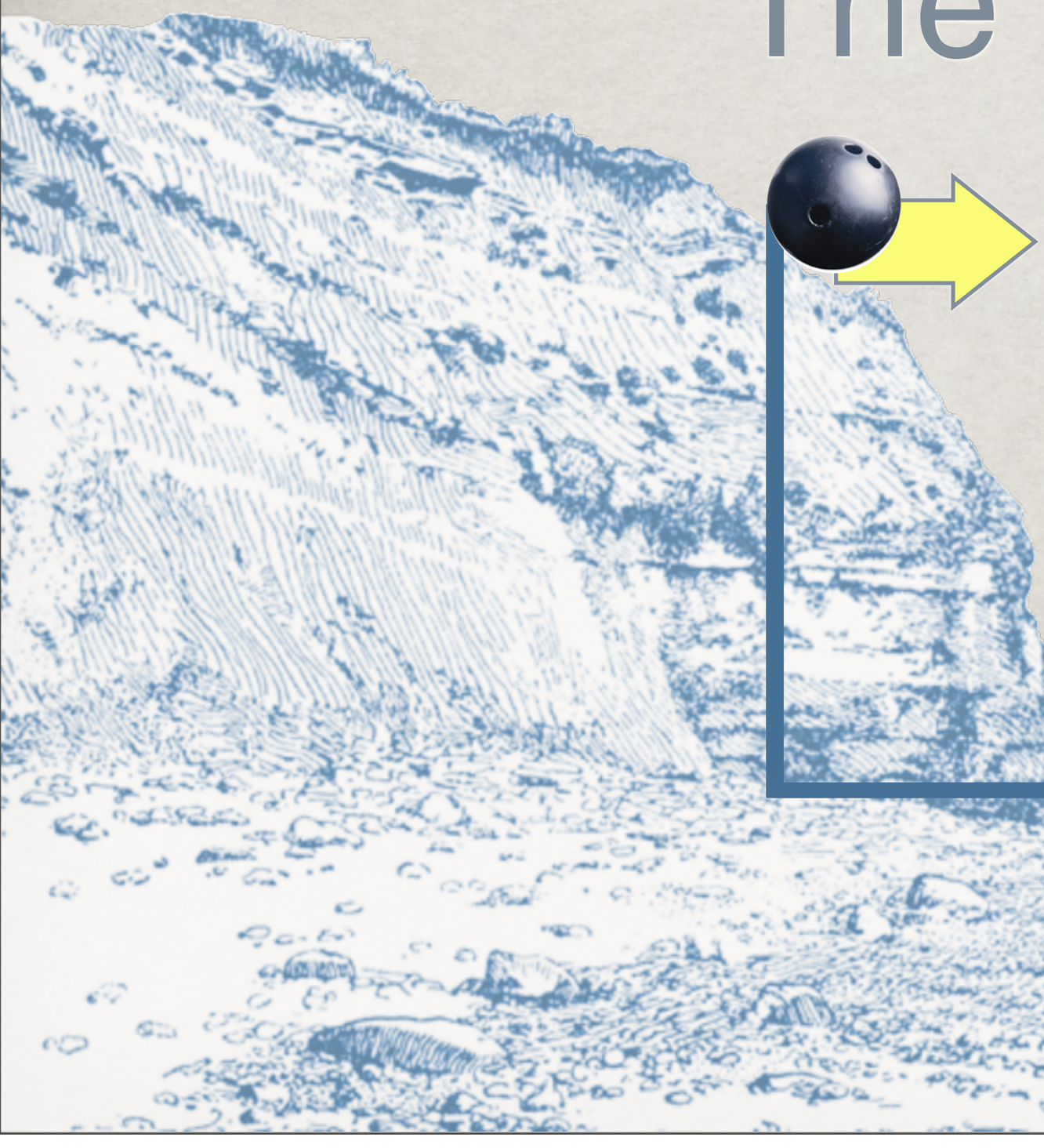
Solve:
Time at the Bottom

- $y_f = y_i + v_y t + \frac{1}{2} a t^2$
- $0 = 101.5 + \frac{1}{2} (-9.8)t^2$
- $t = 4.55s$
- Total Time = 6.08s

Definitions:

- **Simple Projectile Motion:**
  - The motion of a body thrown or fired with an initial velocity v_o in a gravitational field.
- **Projectile:**
  - A kinematic object whose motion is influenced by only the force of gravity.
- **Trajectory:**
  - The path through space followed by a projectile.

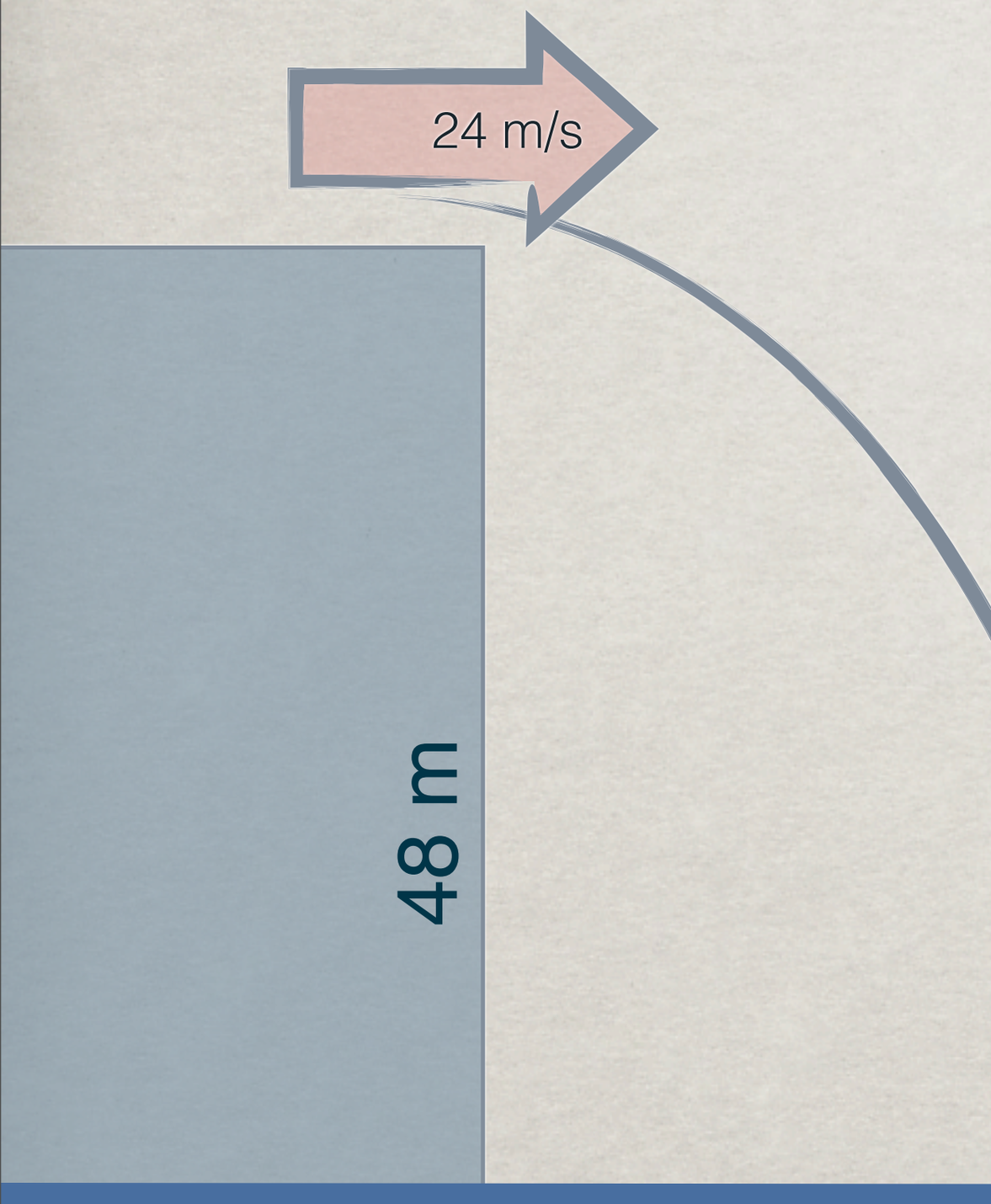
The Cliff Problem



The Cliff

- Problems of this style have an Initial Velocity that is Horizontal
- “x” Velocity is constant
- Common Questions;
  - Find Time
  - Find Range
  - Find Final “y” Velocity
  - Find Final Velocity

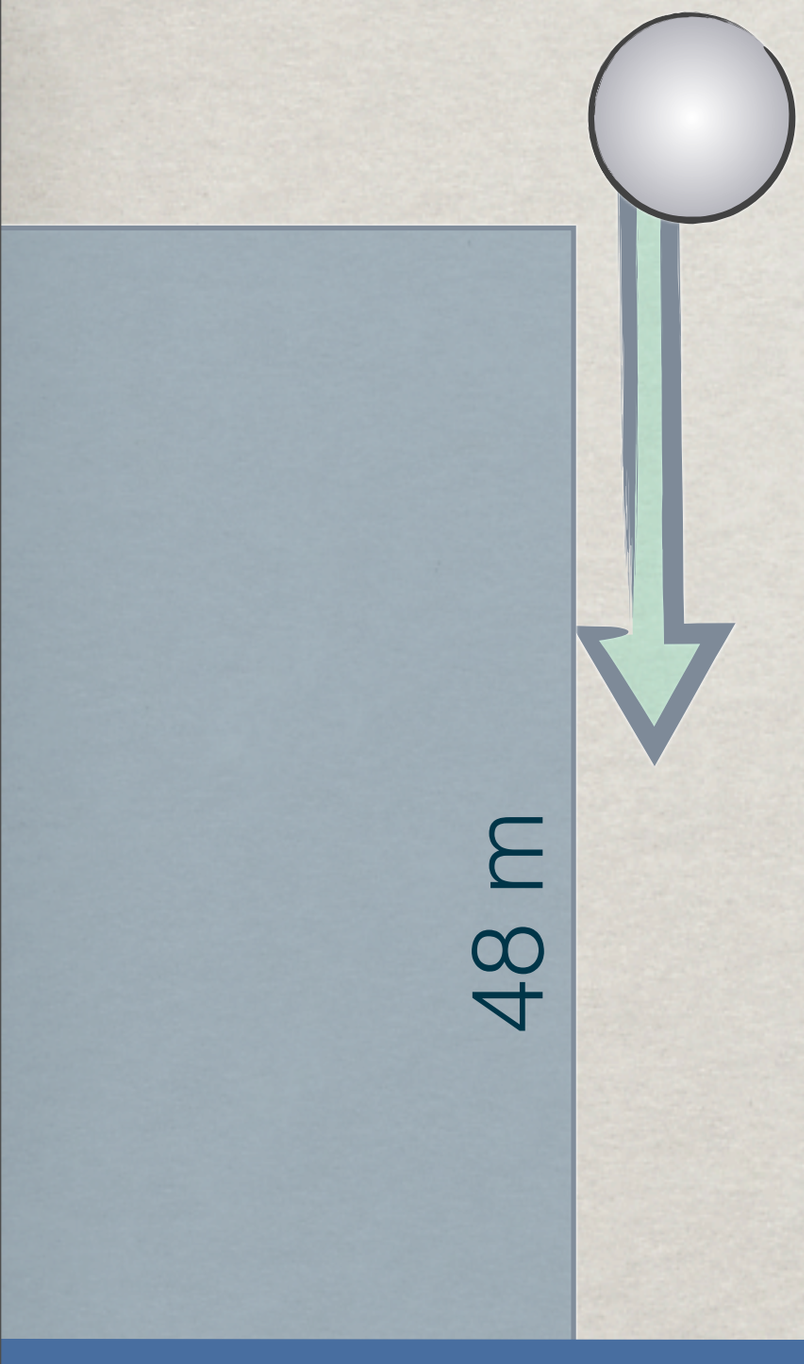
A Ball Rolls From a Cliff



* v_i = 24 m/s
* y_i = 48 m
* Common Questions;
  * Where and when does it land?
  * Final velocity?

Ignore the “complicated” parabola

Find the time to land



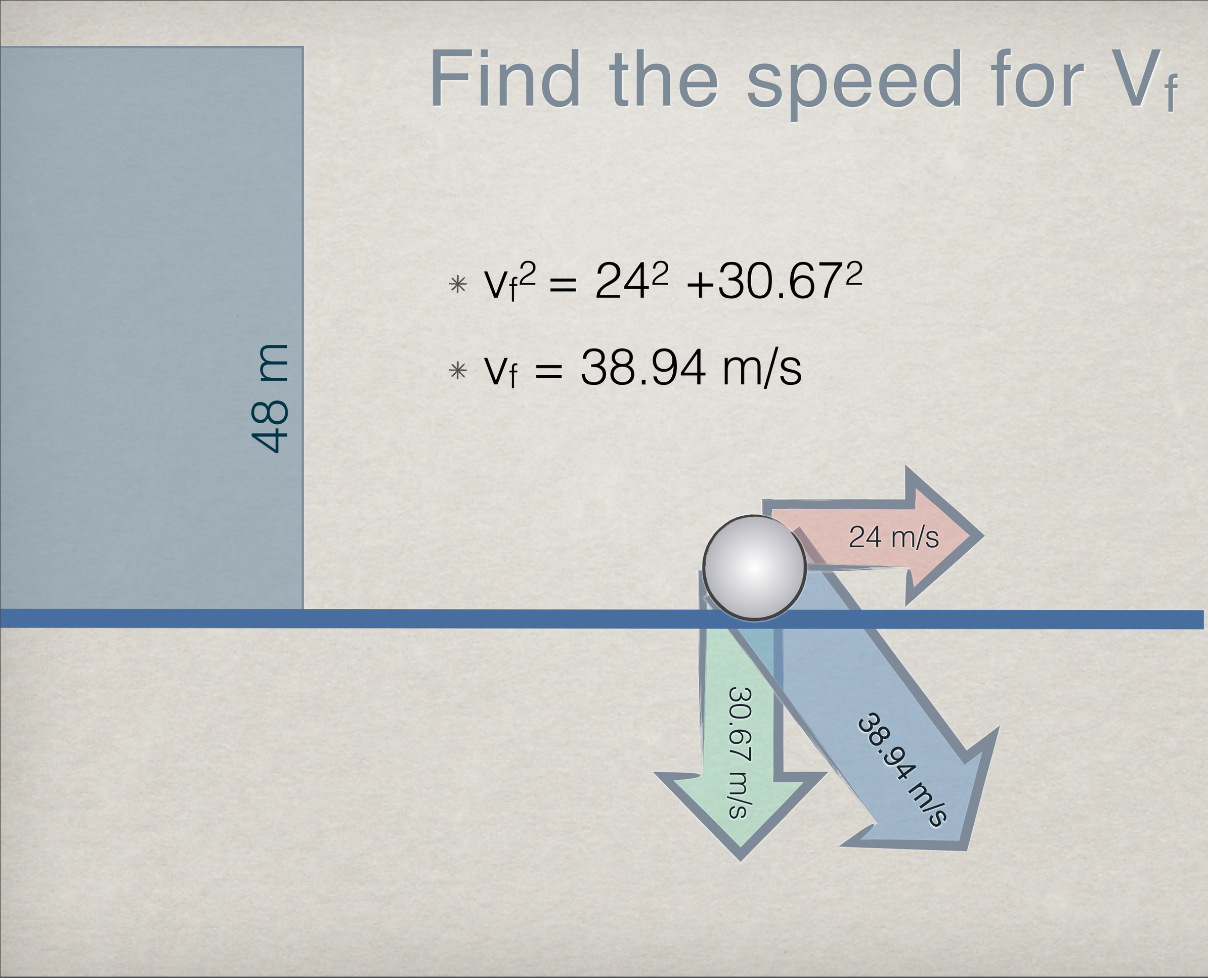
* $y_f = y_i + v_y t + ½ a t^2$
* $0 = 48 + 0 + ½ (-9.8) t^2$
* $48 \div 4.9 = t^2$
* $t = 3.13$ s

Find V_{fy}

* $v_f^2 = v_i^2 + 2\ a\ d$
* $v_{fy}^2 = 0 + 2\ (-9.8)\ (-48)$
* $v_{fy}^2 = 940.8$
* $v_{fy} = \pm 30.67$ m/s

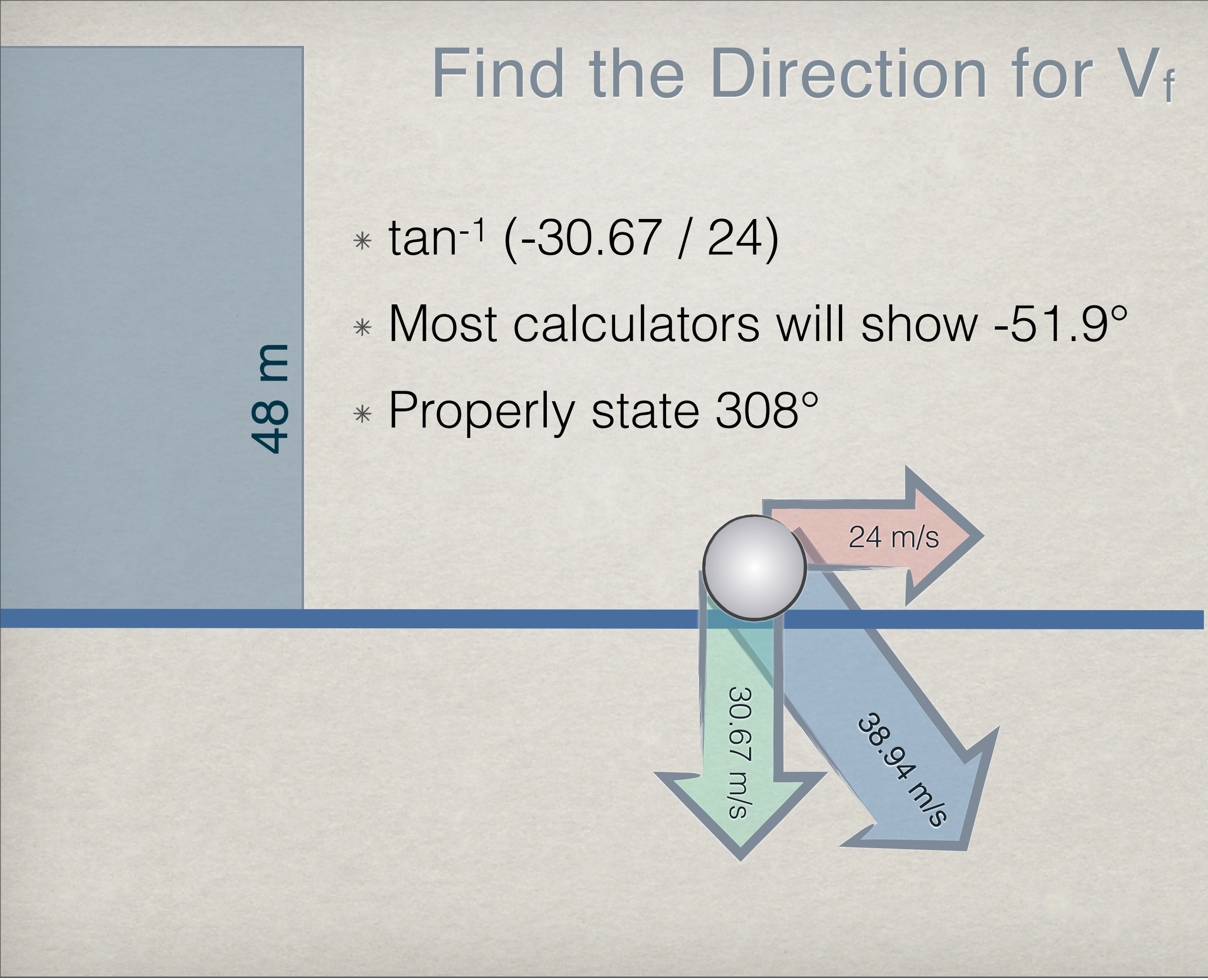
Find the speed for V_f

- $v_f^2 = 24^2 + 30.67^2$
- $v_f = 38.94$ m/s

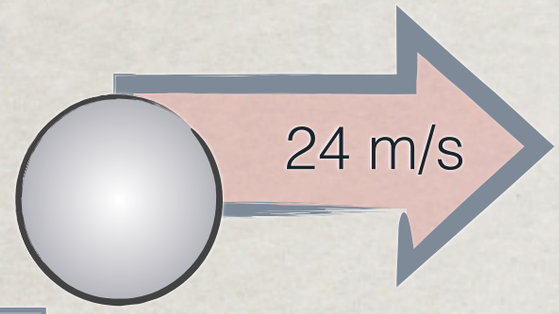


Find the Direction for V_f

- $\tan^{-1}$ (-30.67 / 24)
- Most calculators will show -51.9°
- Properly state 308°



Find the Range

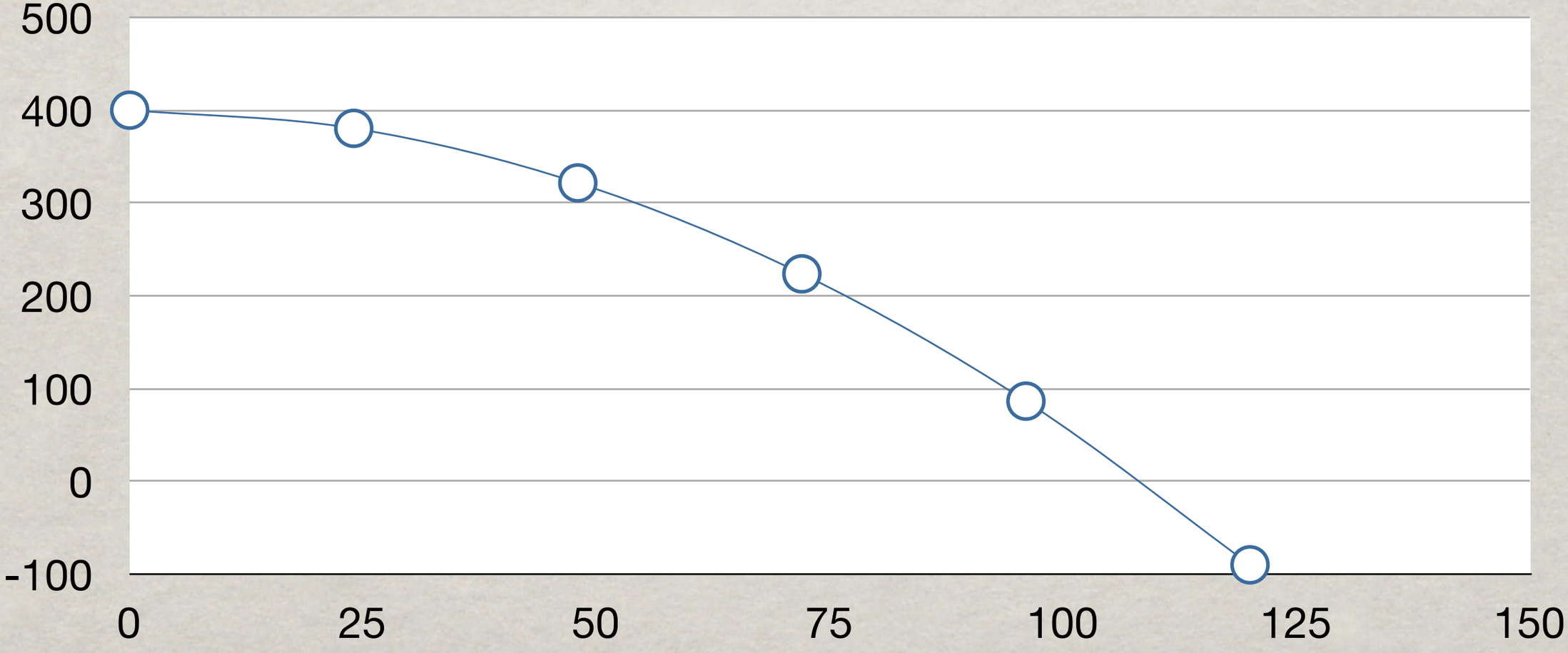


48 m

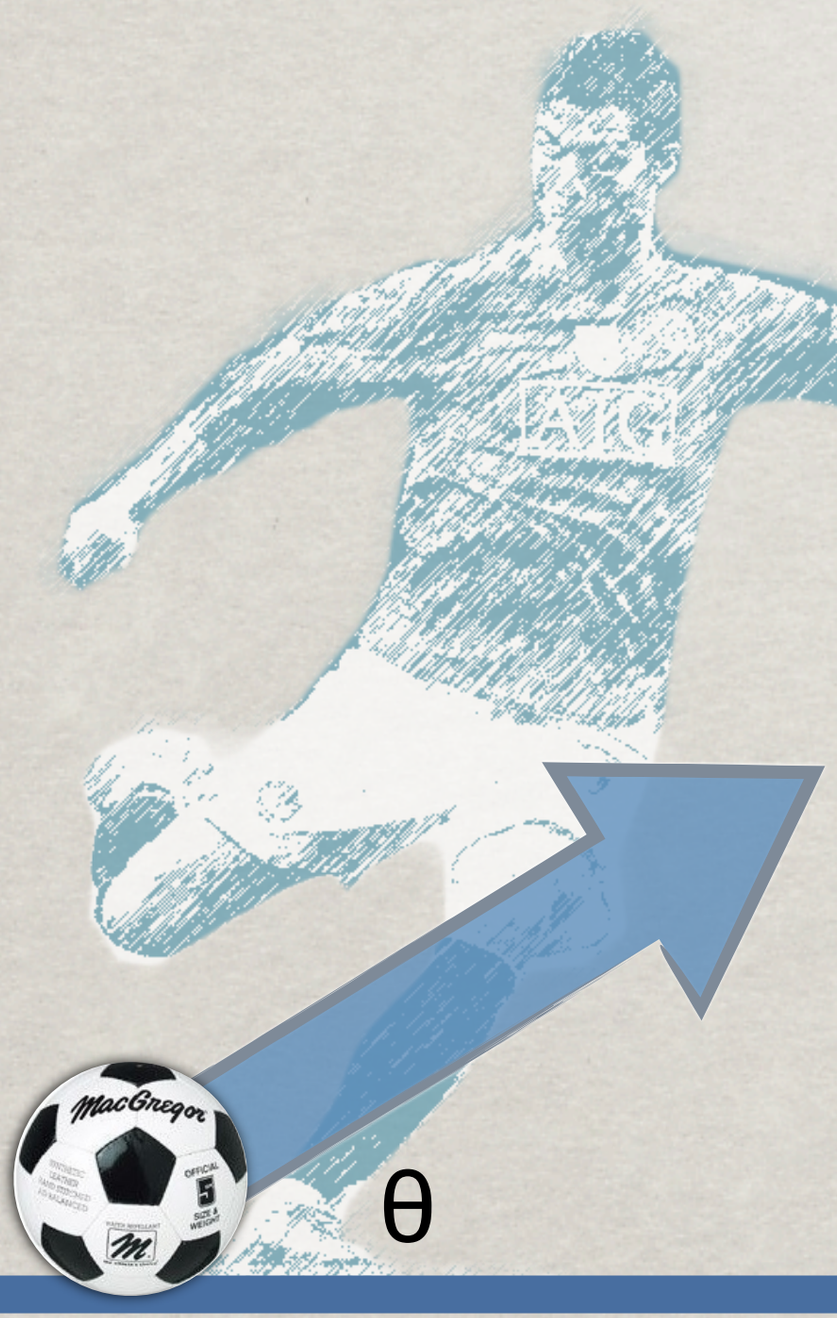
* The distance in the x *when* it lands
* $x_f = x_i + v_x t + \frac{1}{2} a t^2$
    * $t = 3.13$ s
* $x_f = 0 + (24)(3.13) + 0$
* $x_f = 75.12$ m

Location every 2 seconds

time	x velocity	y velocity	x position	y posititon
0	12	0	0	400
2	12	-19.6	24	380.4
4	12	-39.2	48	321.6
6	12	-58.8	72	223.6
8	12	-78.4	96	86.4
10	12	-98	120	-90

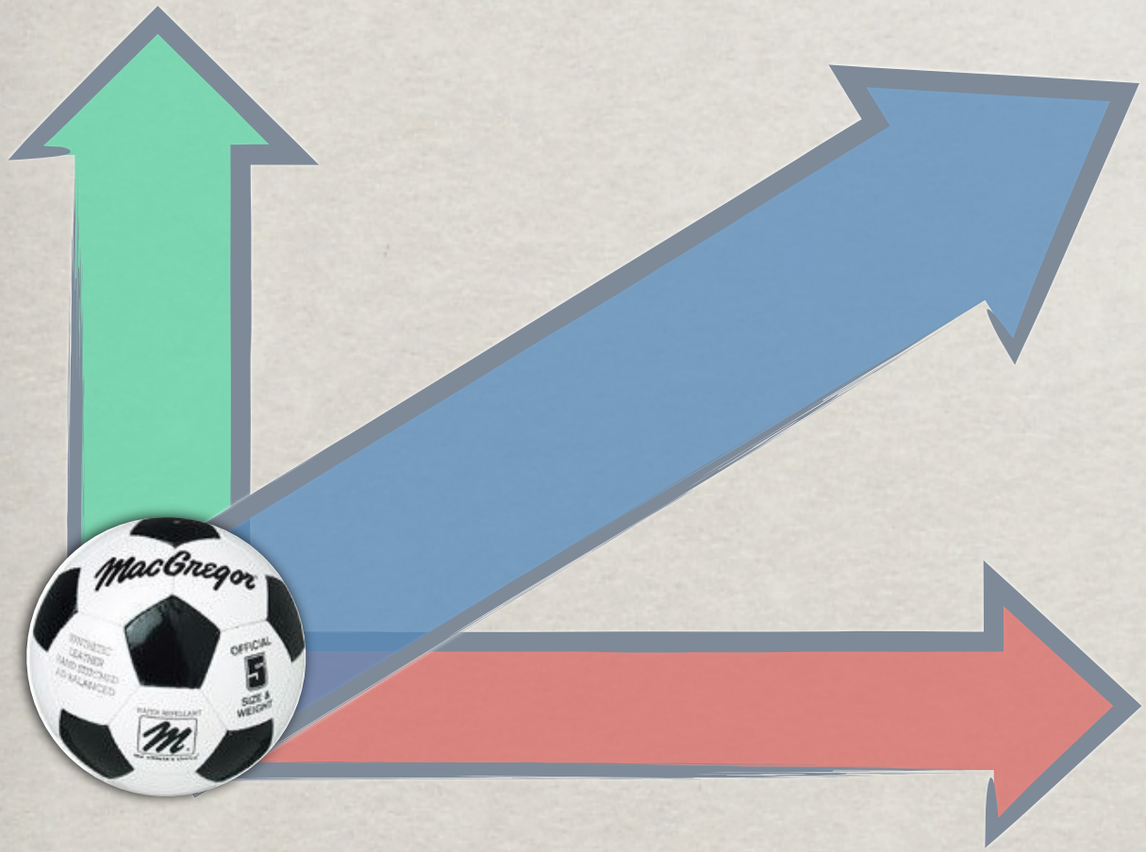


SOCCER BALL



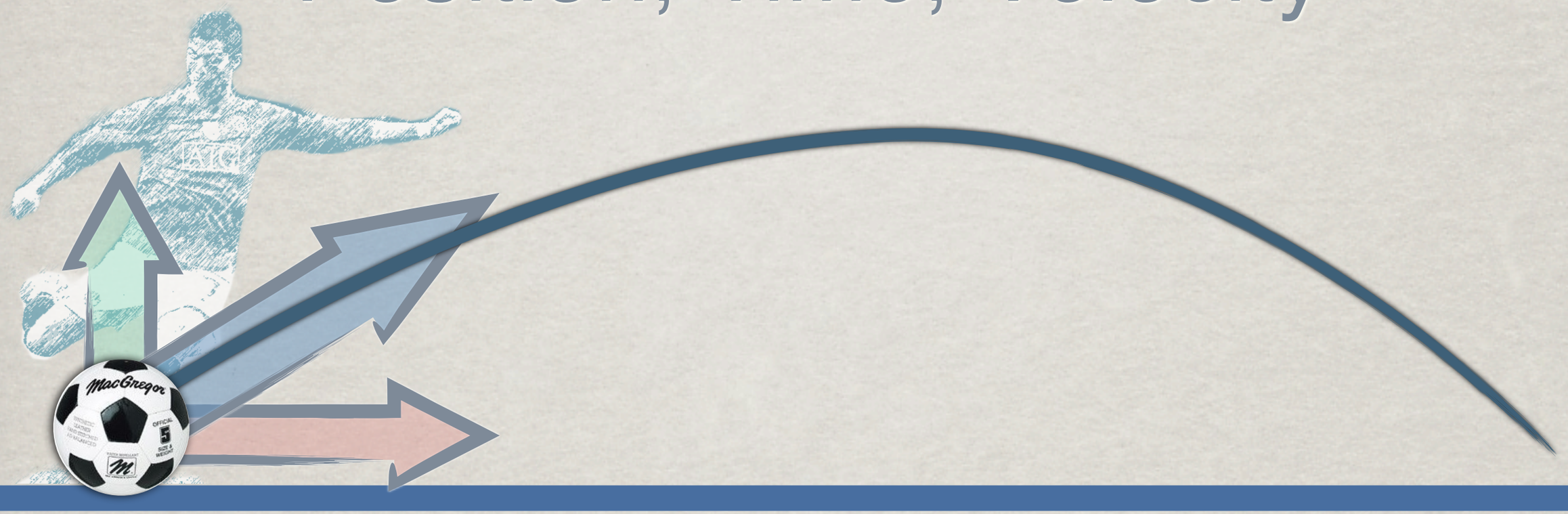
* Initial angle θ for velocity
* Final and Initial Heights are equal

VECTORS



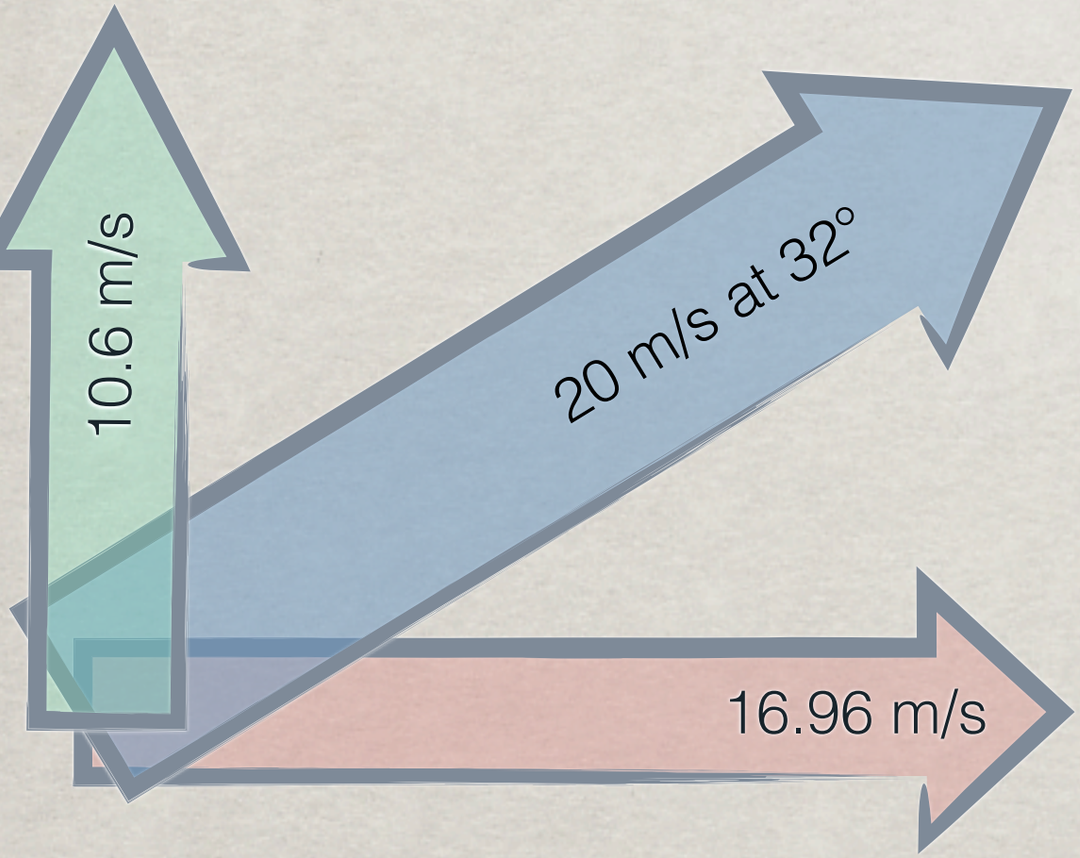
- As a vector at the start of a problem
- X velocity will not change
- Y velocity is changed by gravity
- As a vector at the end of a problem

The Soccer Ball
Position, Time, Velocity



- Common Questions;
  - Maximum Height
  - Where and when does it land?
  - Final velocity?

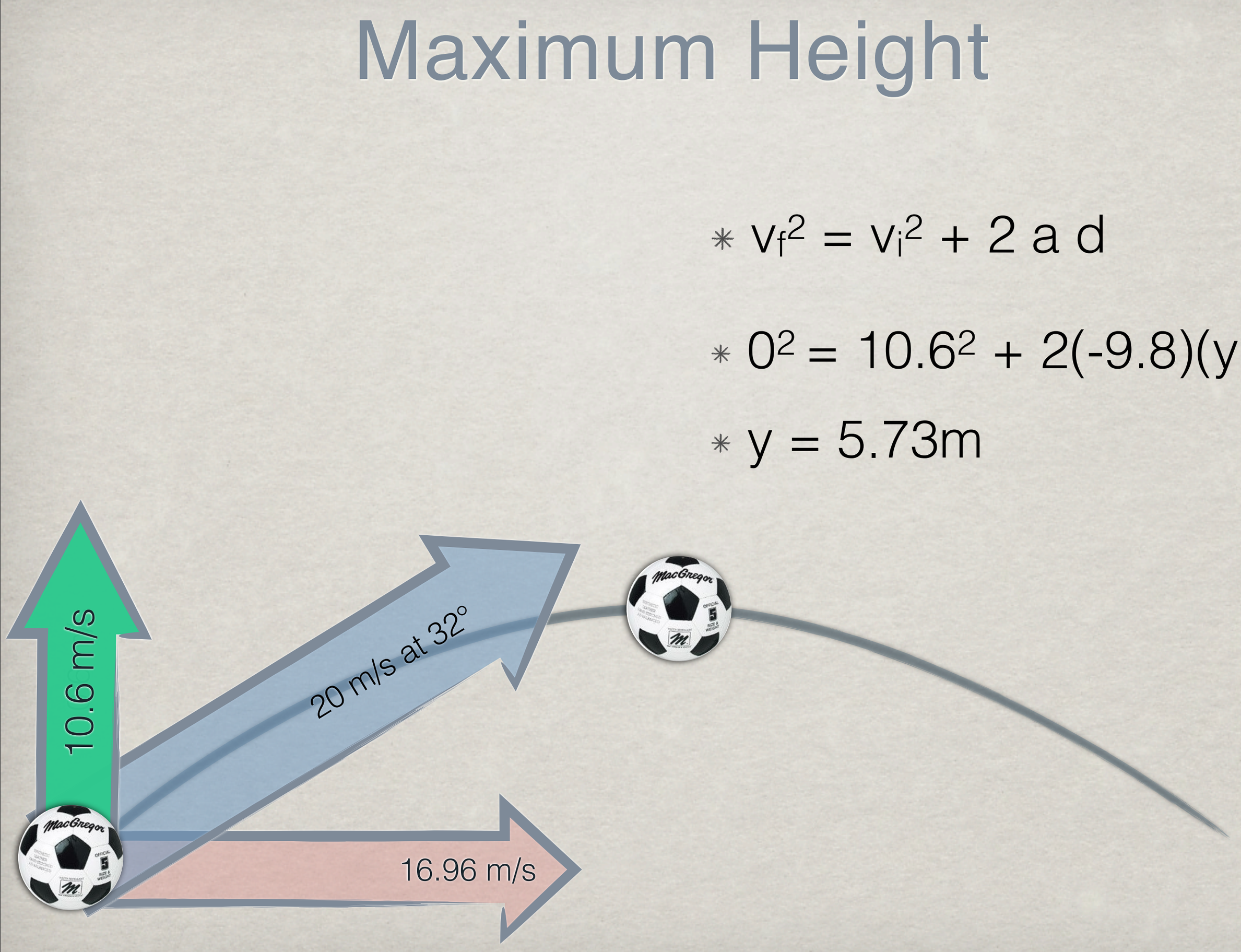
Initial Velocities



* v_i = 20 m/s at 32°
* v_{ix} = 20 cos 32°
* v_{iy} = 20 sin 32°
* a_y = -9.8 m/s^2
* a_x = 0 m/s^2

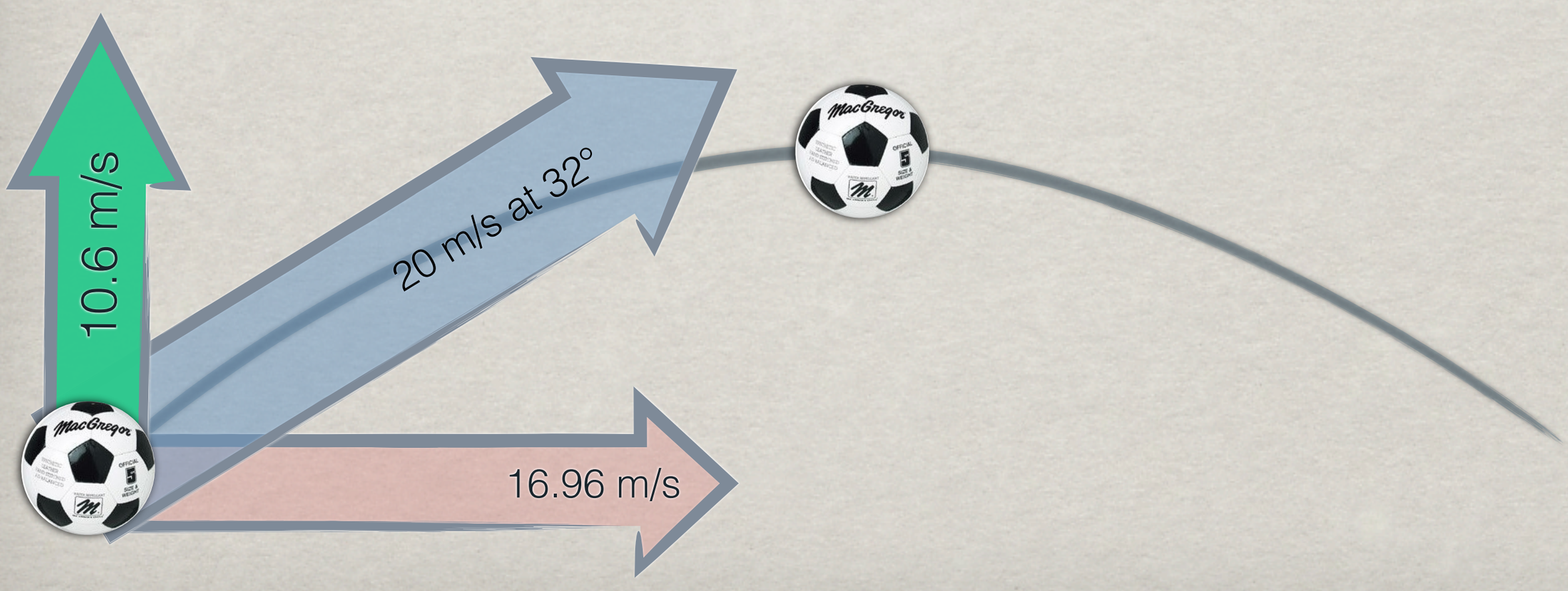
Maximum Height

- $v_f^2 = v_i^2 + 2\ a\ d$
- $0^2 = 10.6^2 + 2(-9.8)(y)$
- $y = 5.73m$



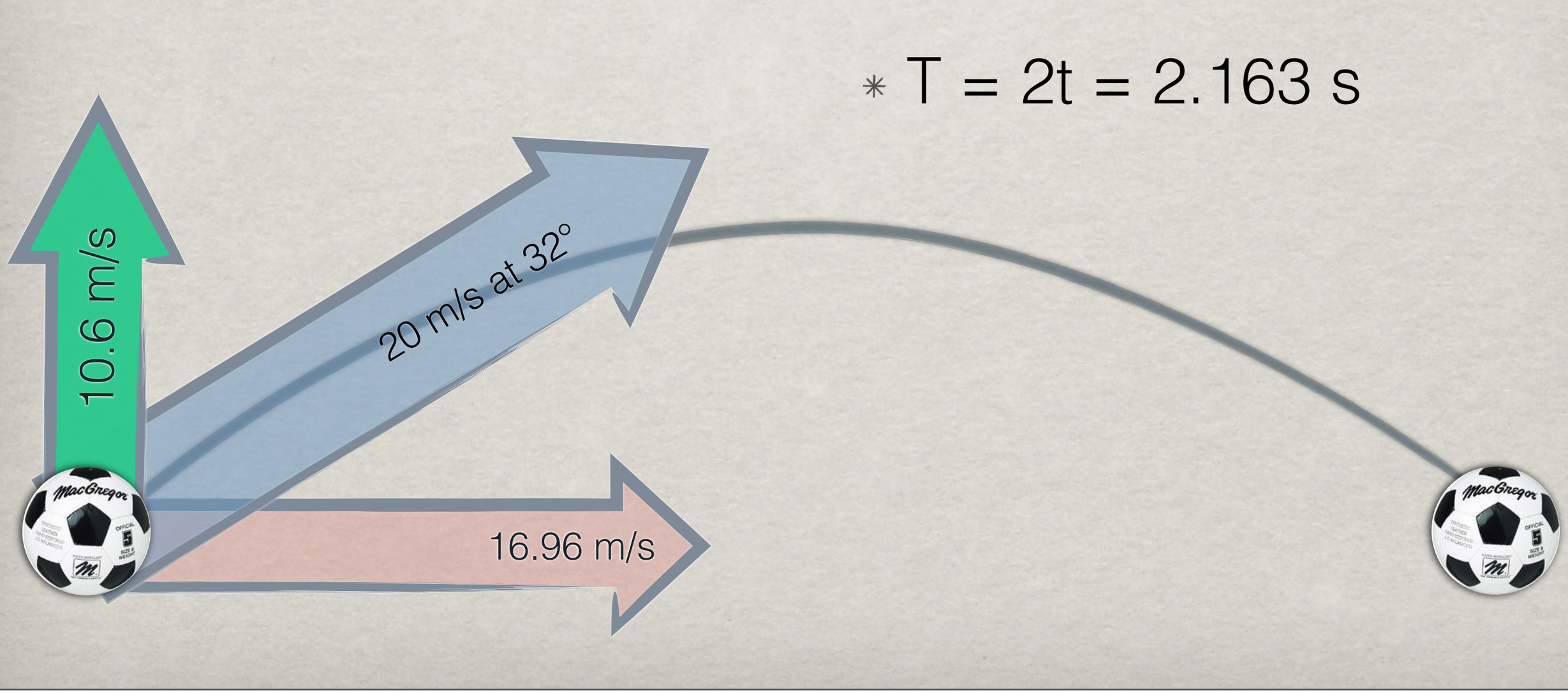
Time at the Top

- $v_f = v_i + at$
- $0 = 10.6 + (-9.8)t$
- $t = 1.08$ s



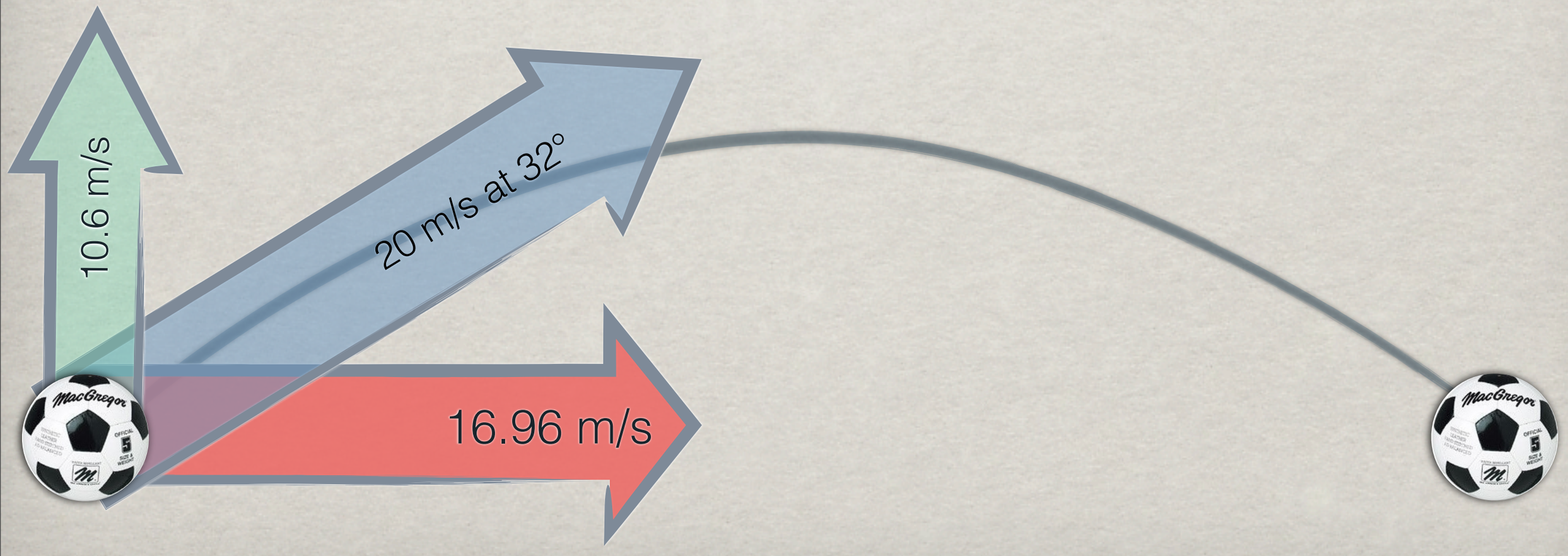
Time to the ground

- t_{top}= 1.08 s
- same distance, and acceleration?
- T = 2t = 2.163 s

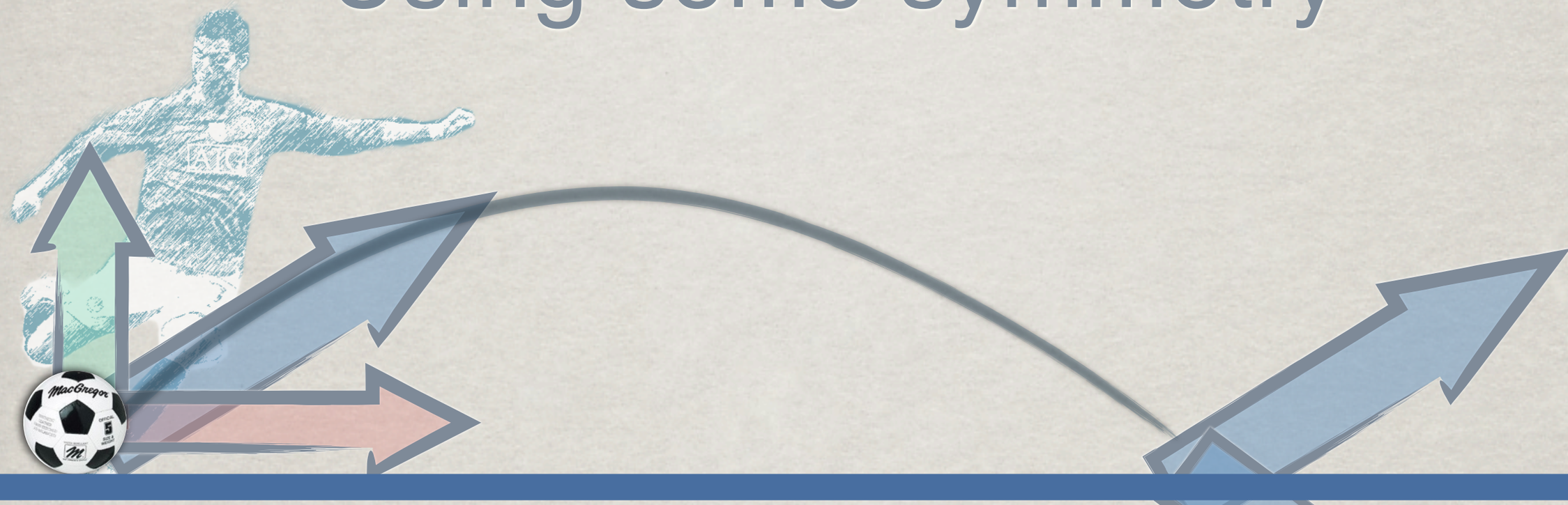


Range - the final x position

- T = 2.163 s
- $x = x + vT + \frac{1}{2} aT^2$
- x = 0 + (16.96)(2.16) + 0
- x_f = 36.7 m



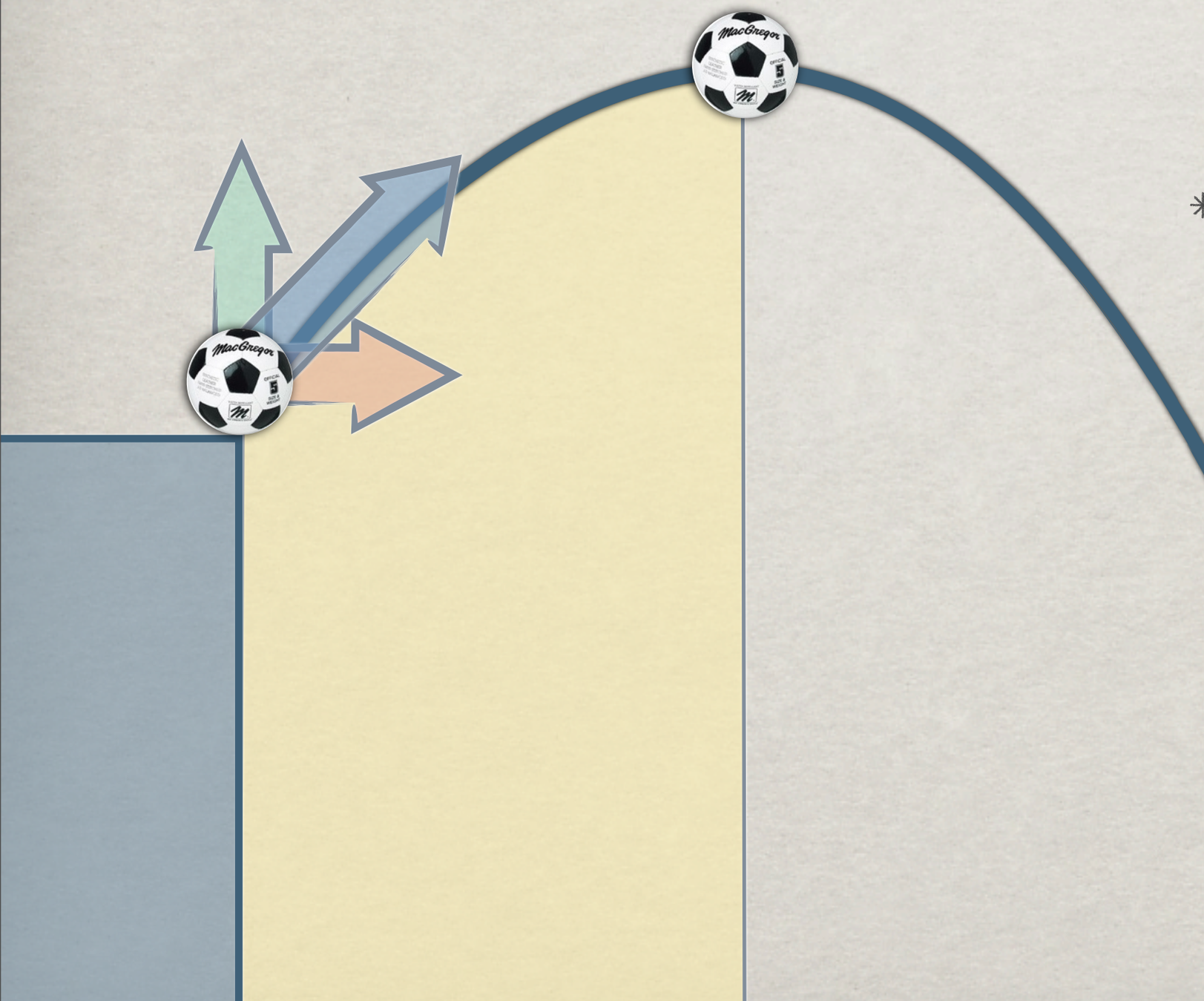
Final Velocity
Using some symmetry



* v_i = 20 m/s at 32°
* v_f = 20 m/s at -32°
* v_f = 20 m/s at 328°

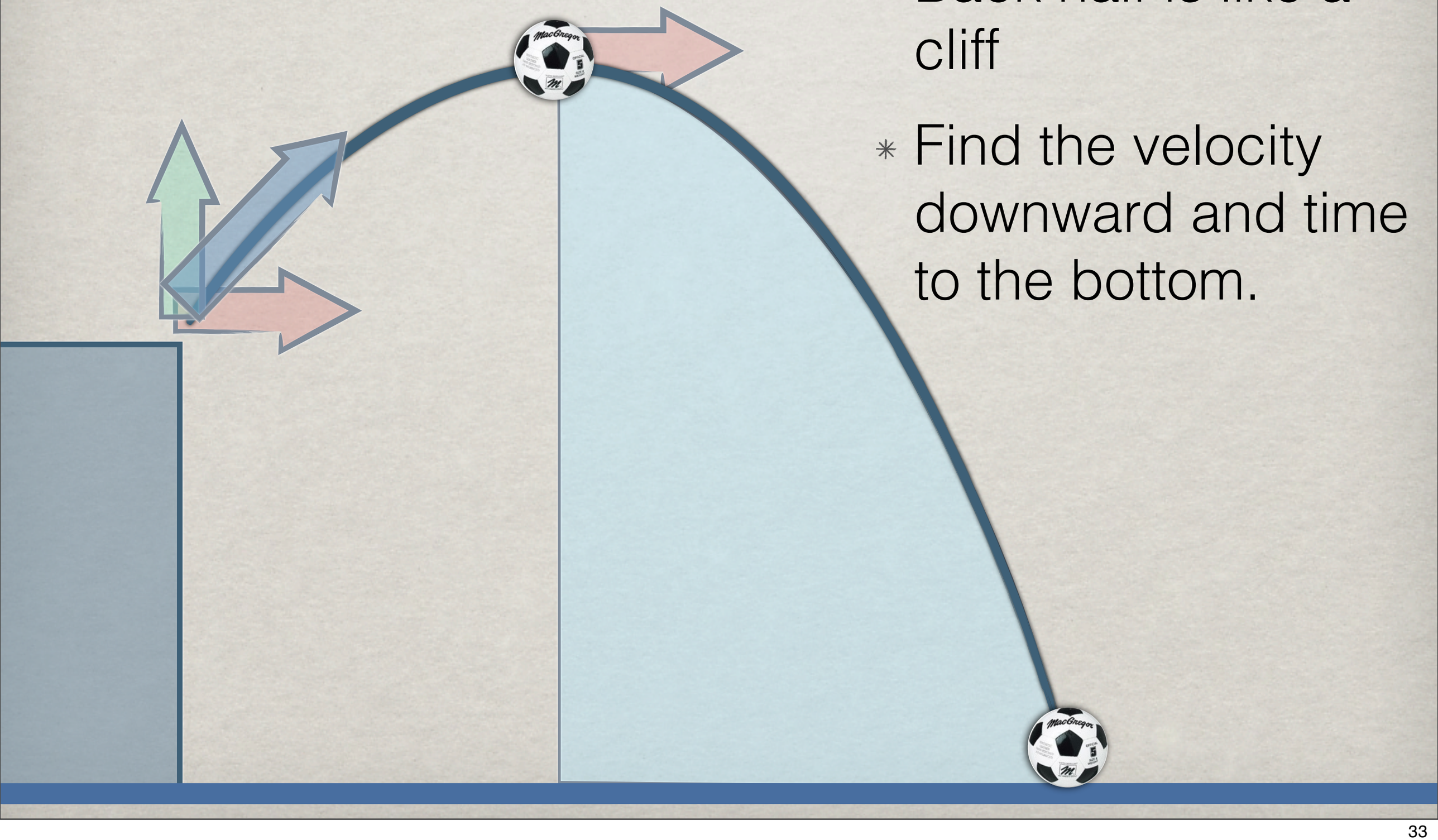
The Combination

Soccer Style



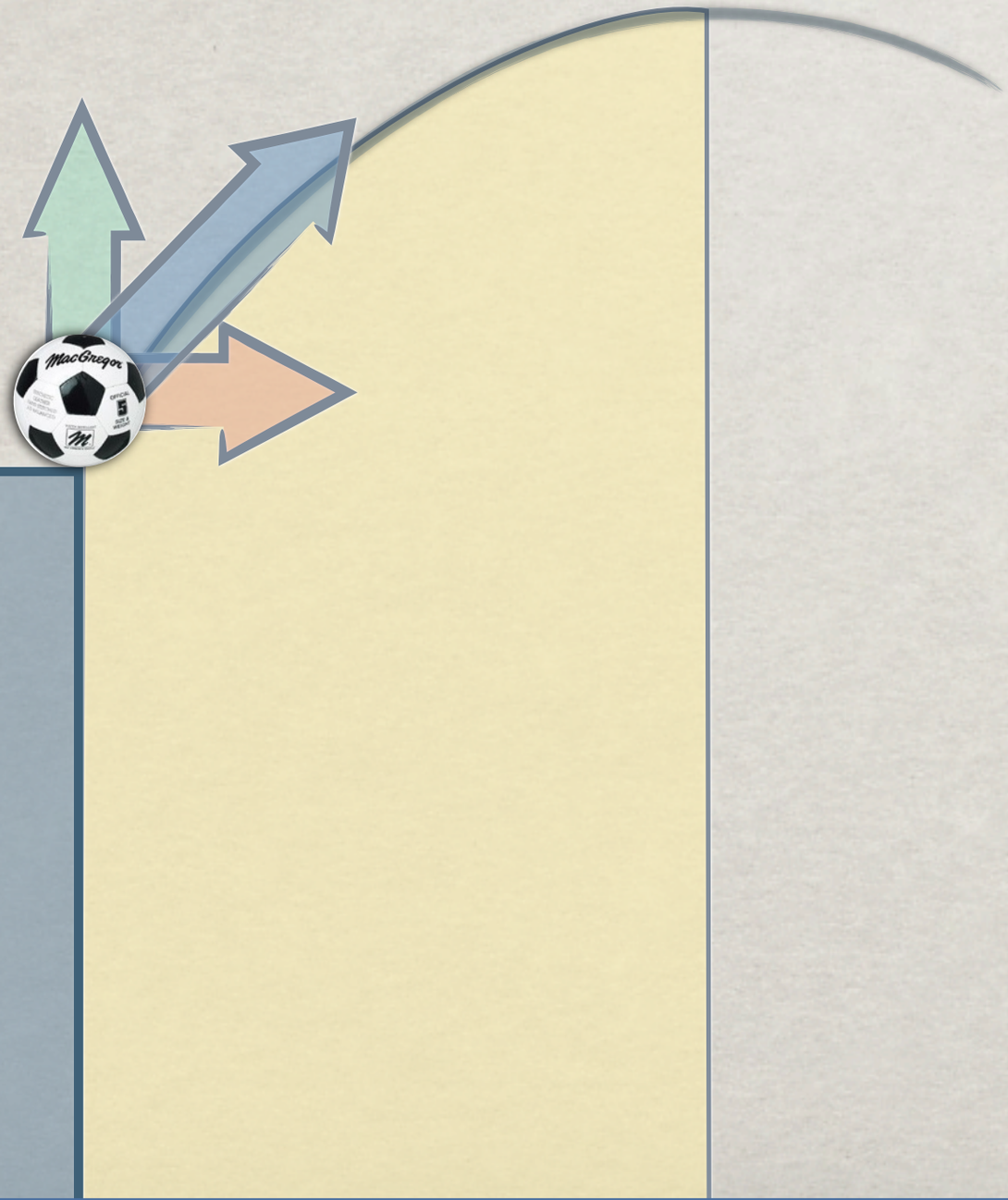
- Front Half is like a soccer problem to the top.
- Find the height and time to the top first

Cliff Style



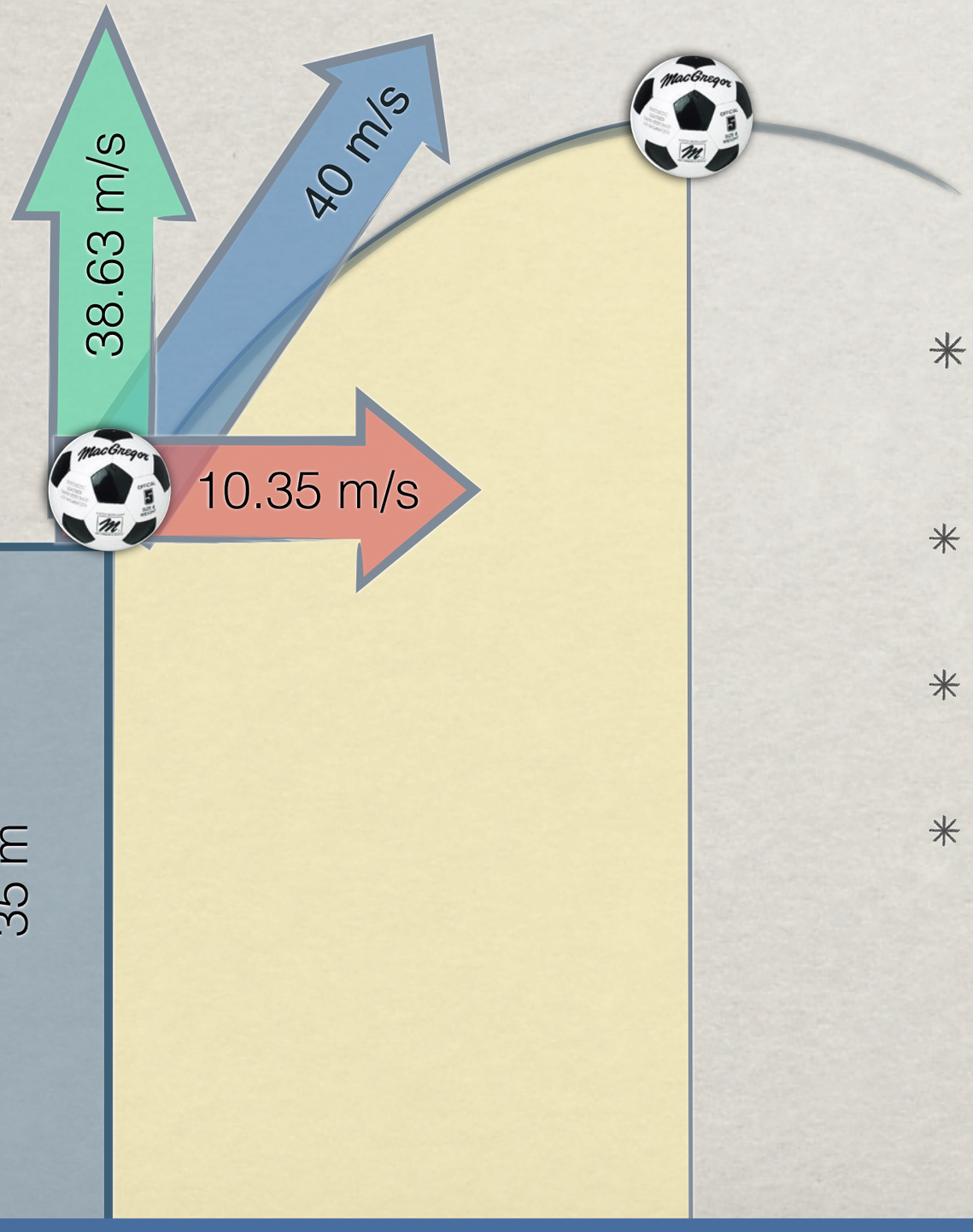
* Back half is like a cliff
* Find the velocity downward and time to the bottom.

Given Information



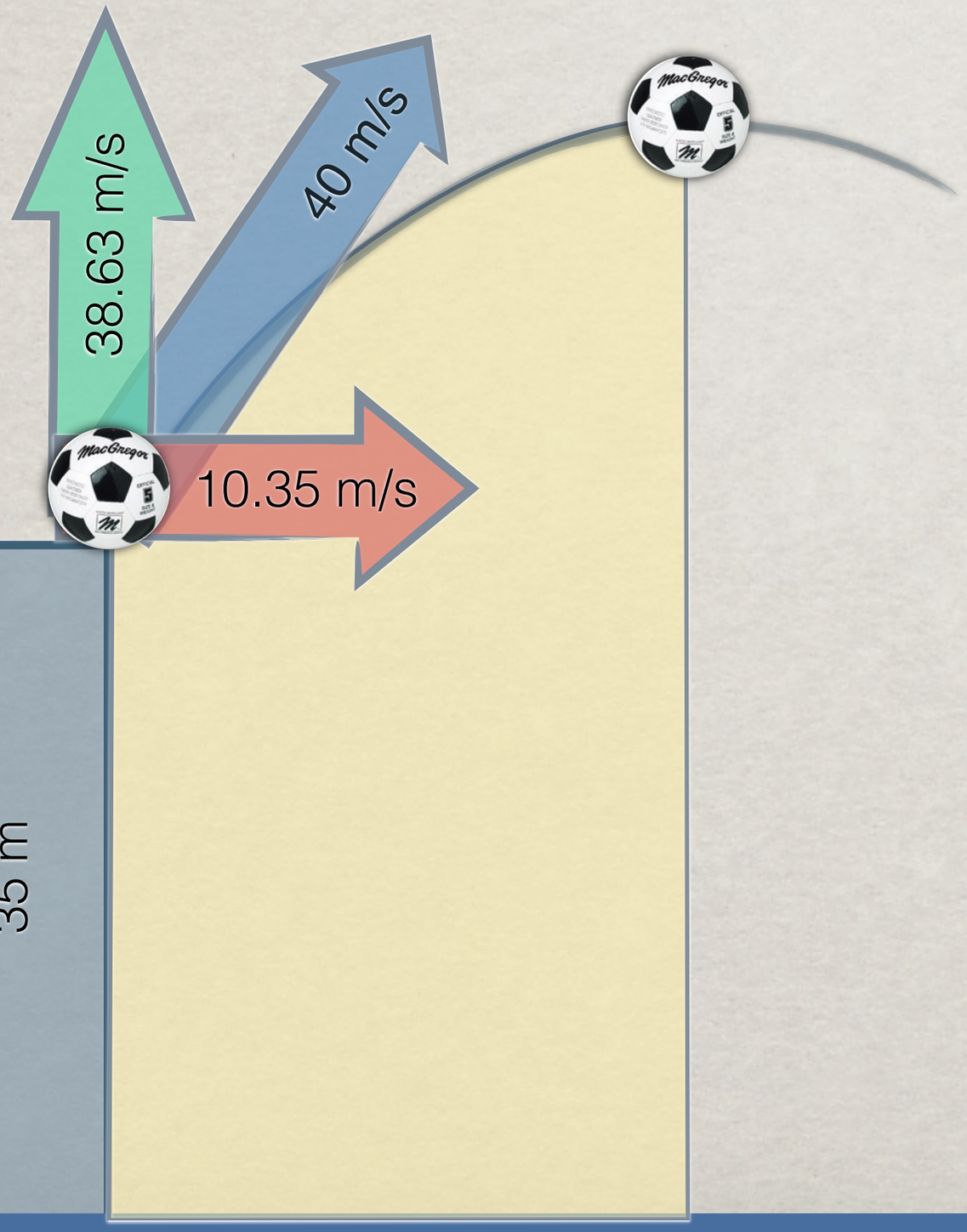
* v_i = 40 m/s at 75°
* y_i = 35 m
* $v_{ix} = 40 \cos 75°$
  * v_{ix} = 10.35 m/s
* $v_{iy} = 40 \sin 75°$
  * v_{iy} = 38.63 m/s

At the Top of the Trajectory



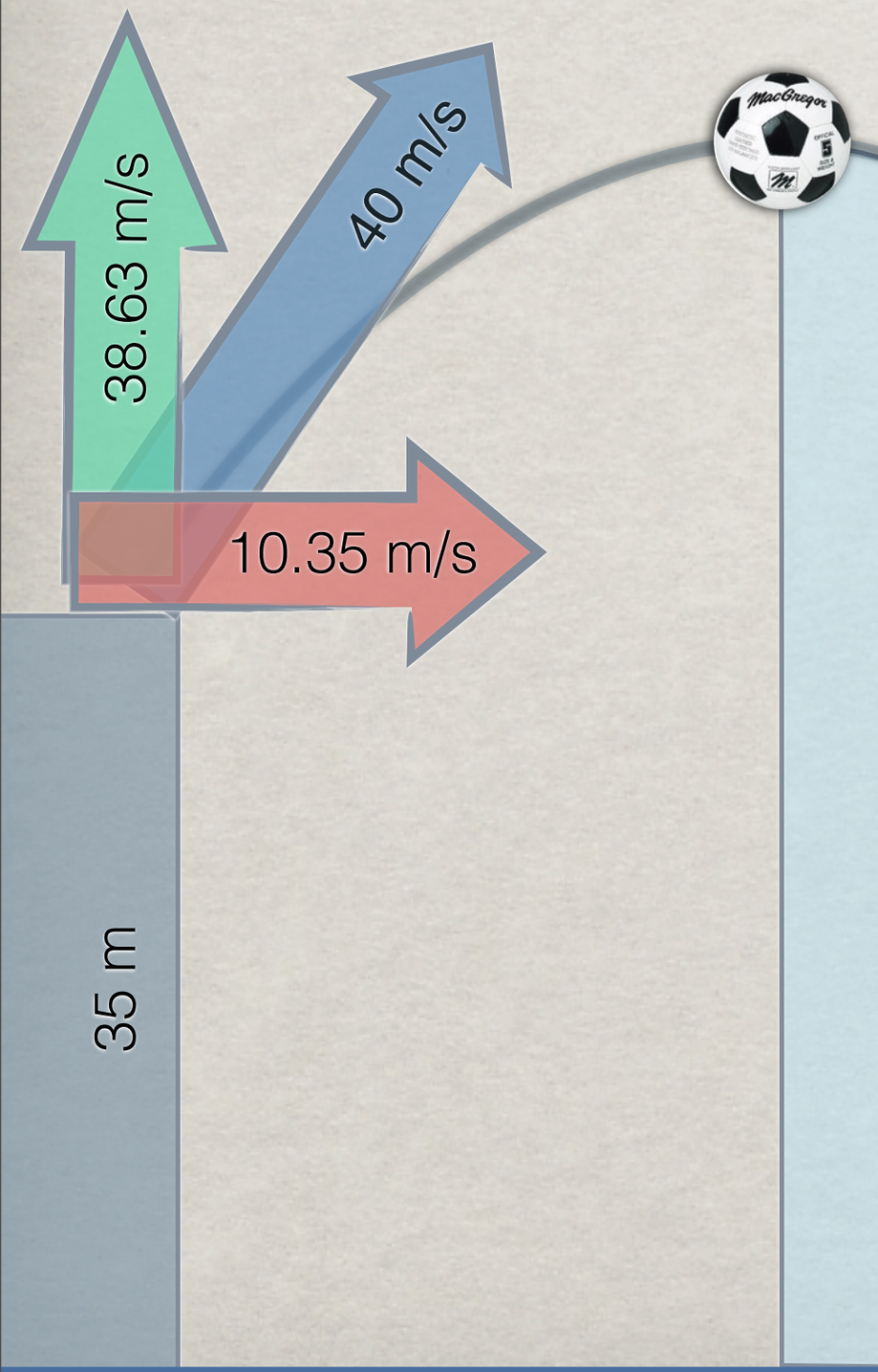
* $v_f^2 = v_i^2 + 2\,a\,d$
* $0^2 = 38.63^2 + 2(-9.8)(d)$
* $d = 76.14\text{m}$
* $y_{max} = h = 111.14\text{ m}$

Time to the Top



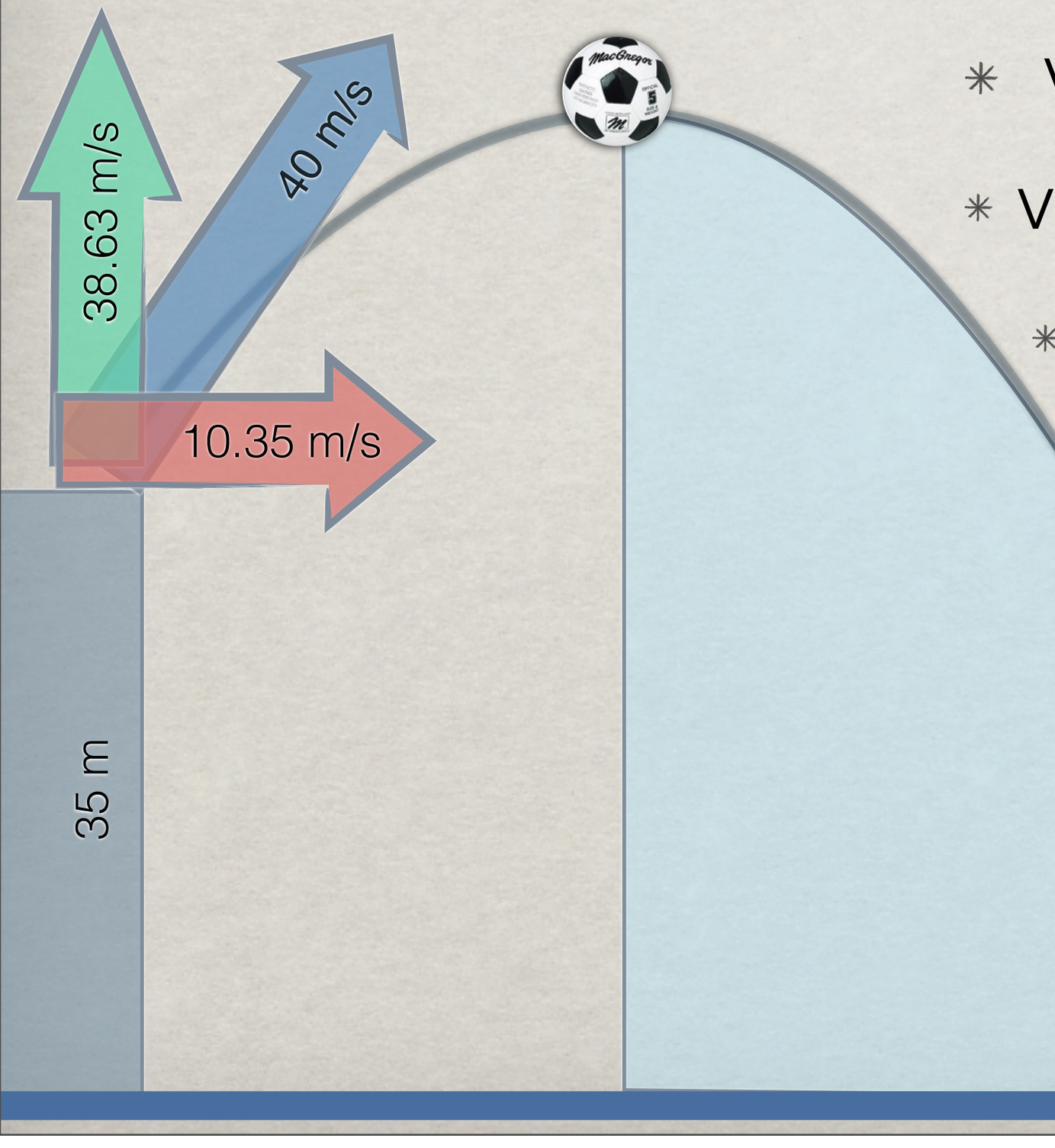
* $v_f = v_i + at$
* $0 = 38.63 + (-9.8)t$
* $t = 3.94$ s

Time to Reach the Ground



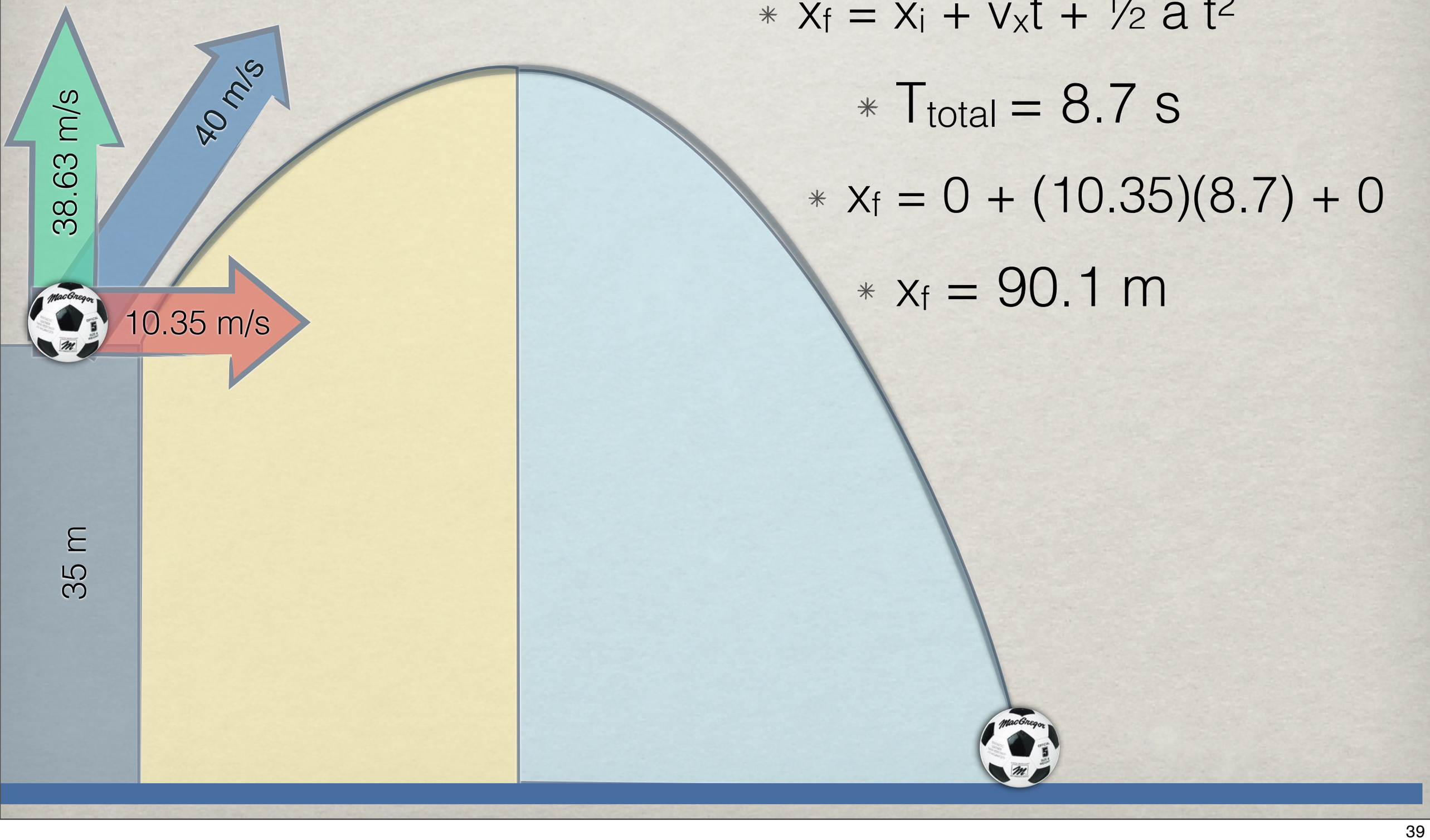
* $y_f = y_i + v_y t + \frac{1}{2} a t^2$
* $0 = 111.14 + 0 + \frac{1}{2}(-9.8)t^2$
  * $t^2 = 111.14 / 4.9$
  * $t = 4.76$ s
* $T_{total} = 8.7$ s

Downward Velocity at the Ground



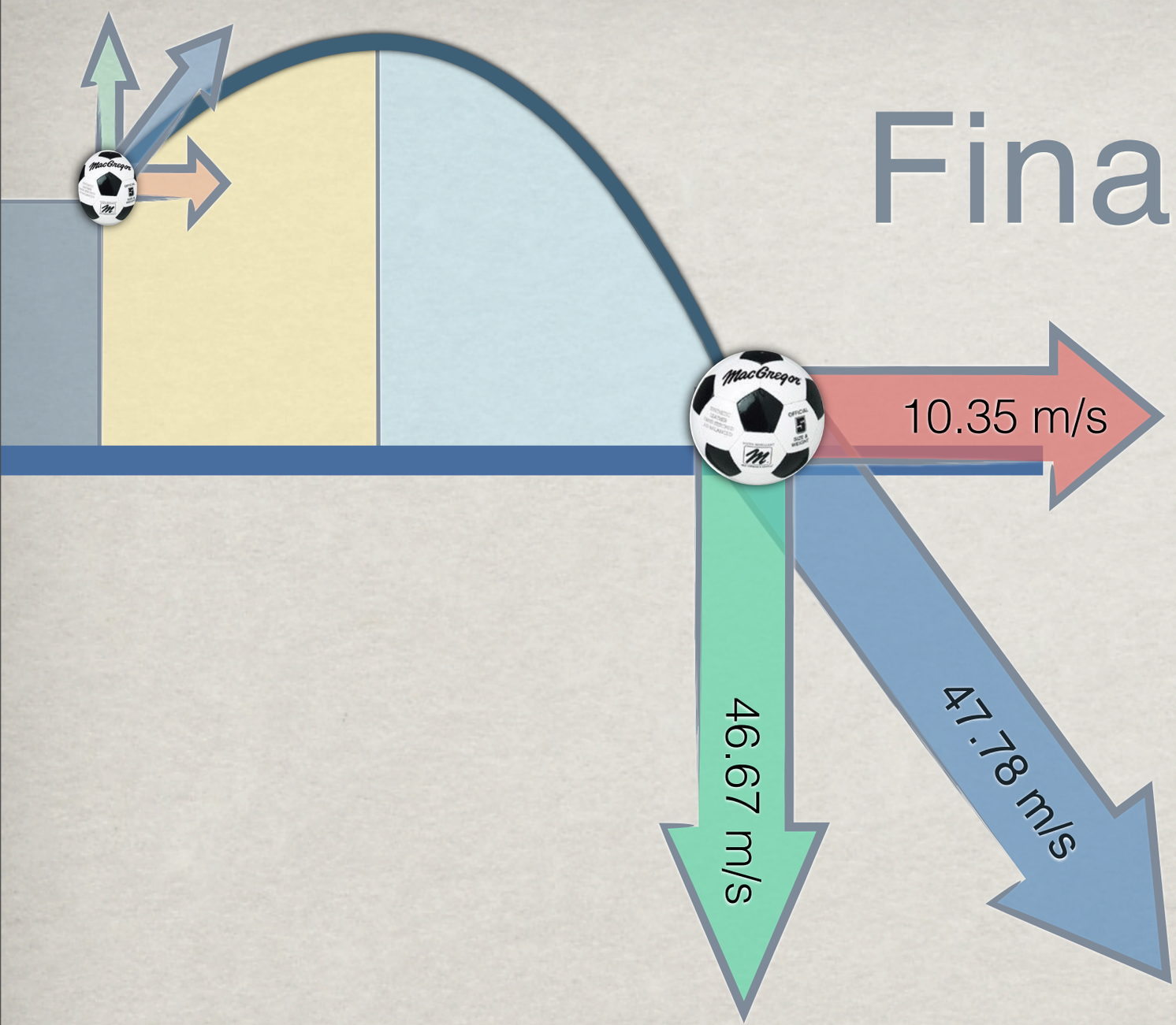
* $v_f^2 = v_i^2 + 2\,a\,d$
* $v_{fy}^2 = 0 + 2(-9.8)(-111.14)$
* $v_{fy}^2 = 2178.3$
* $v_{fy} = -46.67$ m/s

Range



* $x_f = x_i + v_X t + \frac{1}{2} a t^2$
* $T_{total} = 8.7$ s
* $x_f = 0 + (10.35)(8.7) + 0$
* $x_f = 90.1$ m

Final Velocity



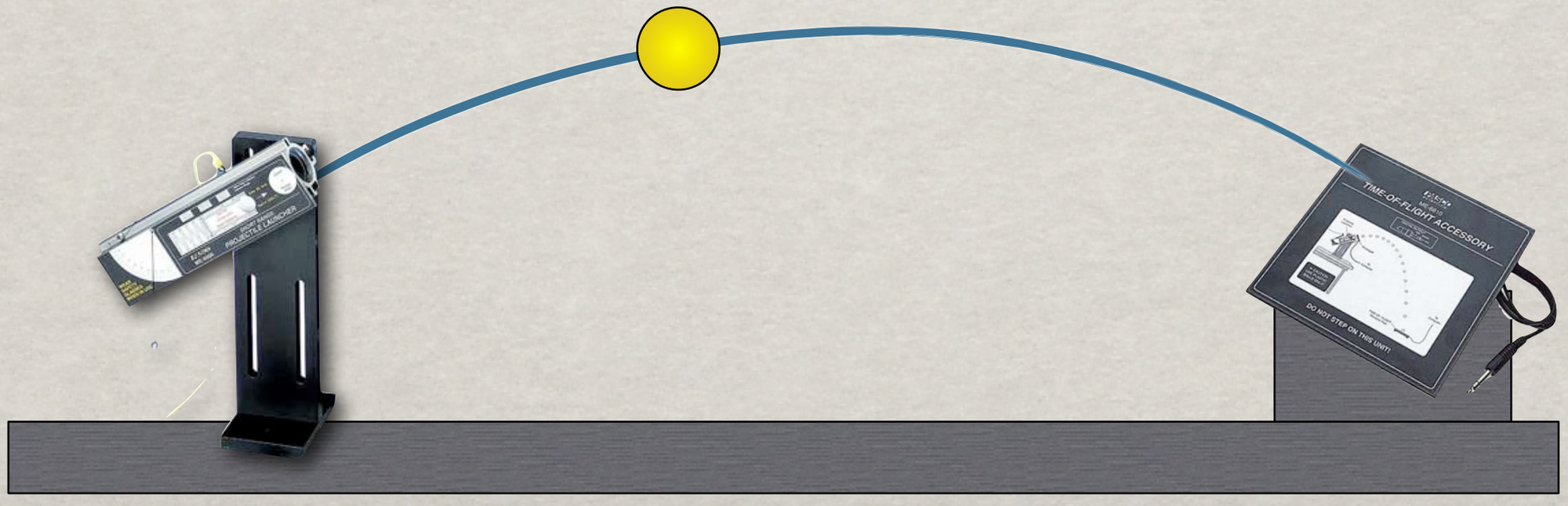
- V_f = 47.78 m/s at 282.5°
- (do the math!)

Projectile Lab Experiment
Vertical Launch

Setting	Average Maximum Height
1 click	
2 clicks	
3 clicks	

Projectile Lab Experiment
Second Launch

Calculated Velocity	Assigned Angle	Predicted Range	Predicted Time



Projectile Lab Experiment
Third Launch

Calculated Velocity	Assigned Angle	Initial Height	Predicted Range	Predicted Time